2021

PHYSICS — HONOURS

First Paper

Full Marks : 100

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from ***each*** **Unit**.

1. Answer ***any ten*** questions : 2×10

(a) Sketch the two Gaussian probability density functions $f_1(x)$ and $f_2(x)$ with the same mean $x = 0$, but with two different standard deviations σ_1 and σ_2, with $\sigma_2 > \sigma_1$.

(b) Prove that $\oint u\vec{\nabla}v \cdot d\vec{r} = -\oint v\vec{\nabla}u \cdot d\vec{r}$.

(c) If two matrices commute, show that they have simultaneous eigenvectors. (Assume the case to be non-degenerate).

(d) Show that $x\delta'(x) = -\,\delta(x)$.

(e) Show that $\delta(ax) = \frac{1}{a}\delta(x)$, where $a > 0$.

(f) State the initial condition of the struck string.

(g) Define linear magnification and angular magnification of an optical system.

(h) The distance between two points in a medium is 3 m. The optical path corresponding to this distance is 4 m. Find out the velocity of light in the medium.

(i) A particle moves with S.H.M. of amplitude 20 cm and period 4 sec. The displacement at $t = 0$ is + 20 cm. Find the position of the particle at $t = 0{\cdot}5$ sec.

(j) What are the characteristics of ideal voltage and current sources?

(k) What is an emitter follower?

(l) Verify the Boolean identity AC + ABC = AC.

Unit - I

2. (a) What is meant by absolute convergence of an infinite series? What is conditionally convergent series? Explain with examples.

(b) For free paths of length x during which a molecule does not suffer a collision with another molecule in a dilute gas, one uses the exponential distribution :

$$P_E(x;\lambda) = \frac{1}{\lambda} e^{-x/\lambda}, \ 0 \le x < \infty$$

Calculate the average value of x in the above distribution. Plot P_E vs. x and calculate the area under the curve.

(c) Four coins are tossed simultaneously. What is the probability of getting at least one head?

4+(2+1+1)+2

3. (a) Verify the divergence theorem for $\vec{A} = 4xz\hat{i} + y^2\hat{j} + yz\hat{k}$ and a cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$.

(b) Prove that $\vec{\nabla} \times \left(\phi\vec{A}\right) = \vec{\nabla}\phi \times \vec{A} + \phi\vec{\nabla} \times \vec{A}$ for any vector $\vec{A}$.

Hence prove that $\oint\left(u\vec{\nabla}v\right)\cdot d\vec{r} = \int_S \left(\vec{\nabla}u\right)\times\left(\vec{\nabla}v\right)\cdot d\vec{S}$ for any two scalars u and v.

6+(2+2)

4. (a) Using divergence theorem, prove that $\oint d\vec{S} = 0$ for any closed surface.

(b) Find the eigenvalues and normalized eigenvectors of the matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

(c) Prove that the commutator of two Hermitian matrices is skew-Hermitian (anti-Hermitian).

(d) Prove that the product of two unitary matrices is also unitary.

2+(2+2)+2+2

5. (a) Consider Hermite's equation :

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2\alpha y = 0$$

and assume a series solution $y(x) = \sum_{\lambda=0}^{\infty} a_\lambda x^{k+\lambda}$.

(i) Find the indicial equation.

(ii) Find the recurrence relation among the coefficients a_λ.

(iiii) Find the condition on α so that the infinite series solution becomes a polynomial.

(b) Solve $\frac{\partial U}{\partial x} = 4\frac{\partial U}{\partial y}$ by the method of separation of variables, given that $U(x, 0) = 8e^{-3x}$.

(2+2+2)+4

6. (a) Laplace's equation in spherical polar coordinates for a problem with azimuthal symmetry is given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)=0 .$$

Let $V(r, \theta) = R(r)T(\theta)$. Taking the separation constant to be $l(l+1)$, solve for $R(r)$. Also, show that the substitution of $w = \cos\theta$ in the angular part leads to Legendre's equation for $T(\theta) = P(w)$.

(b) Show that the Fourier transform of $f(x) = e^{-|x|}$ is $F(k)=\sqrt{\frac{2}{\pi}}\frac{1}{k^2+1}$. (3+3)+4

7. (a) Expand $f(x)=\begin{cases}0, -\pi < x \leq 0\\ x,\ 0 \leq x < \pi\end{cases}$ in a Fourier series.

Hence show that $\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+...$

(b) State the Dirichlet conditions for validity of a Fourier series expansion. Do these conditions hold for the function $\tan x$? Explain. (4+2)+(2+1+1)

Unit - II

8. (a) Using Fermat's principle, deduce the relation $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$ (with usual symbols) for paraxial image formation by a concave mirror.

(b) Explain the term 'optical path'.

(c) What is meant by equivalent lens of two thin lenses separated by a distance? Explain with diagram. 5+2+3

9. (a) Show that in case of forced vibration, $\frac{average\ K.E}{average\ P.E}=\frac{\omega^2}{\omega_0^2}$ where ω_0 is the natural frequency of oscillation and ω is the frequency of the driving system.

(b) The dispersion relation for transverse waves propagating in a medium is given by $\omega^2 = {\omega_p}^2 + k^2c^2$ where ω is angular frequency, k is wave number and ω_p and c are constants. Show that $v_g v_p = c^2$, where v_g is group velocity and v_p is phase velocity.

(c) A particle is subjected to two SHM-s represented by $x = A\cos\omega t$ and $y = B\sin 2\omega t$. Find the equation for the resultant locus in XY plane.

(d) A plane progressive wave is given by $y(x,t)=A\sin\left(\omega t-\frac{\omega}{v}x+\alpha\right)$. Find the differential equation for the wave motion. 3+2+3+2

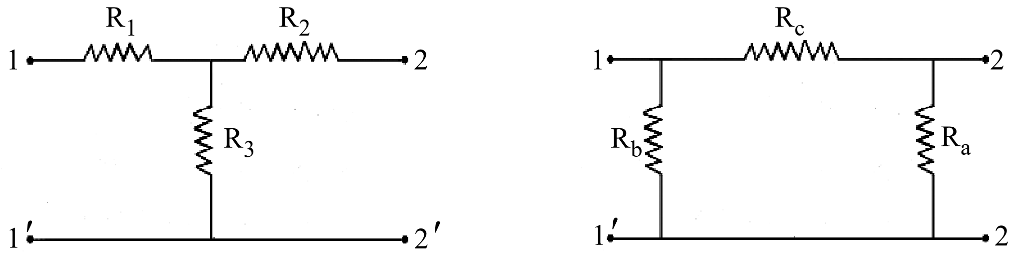
Please Turn Over

10. (a) Explain the terms 'refraction matrix' and 'system matrix' of an optical system for refraction of paraxial rays.

(b) Consider a plano-convex lens of a material of refractive index 1·5. The convex surface has a radius of 5 cm and is facing the incident light. The central thickness of the lens is 3 mm. Obtain the system matrix.

(c) State Huygen's principle. Apply it to deduce the laws of reflection of plane waves at a plane reflector. 4+2+(1+3)

11. (a) Consider the T and π-networks of resistances of the following figures :



Show that these networks will be equivalent in the sense that the resistances between the corresponding pair of terminals will be identical provided

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \quad R_2 = \frac{R_c R_a}{R_a + R_b + R_c}, \quad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

(b) Define reverse saturation current of a p–n junction diode. Why is it temperature dependent?

(c) Make a comparative study of CB, CC and CE amplifiers with reference to current and voltage gain. 4+(2+1)+3

12. (a) State and explain maximum power transfer theorem.

(b) What is the difference between an enhancement and a depletion MOSFET?

(c) Explain the basic principle of an LED.

(d) How does an FET differ from a BJT? (1+3)+2+2+2

13. (a) Using discrete components, draw the circuit diagram of an AND gate and explain how it functions.

(b) Draw diagram and explain how one can obtain the function of the gates OR, AND and NOT, by using NOR gates only.

(c) Verify the Boolean identity $A + \bar{A}B = A + B$. 4+4+2