2021

PHYSICS — HONOURS

Fourth Paper

(Group - A)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Symbols have the usual meanings everywhere]

Answer *question no.* **1** and *any four* questions from the rest.

1. Answer *any five* questions : 2×5

(a) Check whether $\psi(x) = Ae^{-x^2}$ and $\psi(x) = Bxe^{x^2}$ are well-behaved at $-\infty < x < \infty$.

(b) If the uncertainty in the position of a particle be determined up to an accuracy of 10^{-8} m, what is the corresponding uncertainty in momentum?

(c) Under what condition is the product of two Hermitian operators also Hermitian?

(d) Calculate the work done in a quasistatic isothermal expansion of an ideal gas at temperature T where the initial and final pressures are P_1 and P_2 respectively.

(e) Show that in an isobaric process, the change in enthalpy is equal to the heat transferred between the system and the surrounding.

(f) Draw the pressure-temperature phase diagram of H_2O indicating the phases, boundaries and the triple point.

2. (a) Justify the statement that photoelectric effect supports the particle nature of light. Mention one phenomenon which supports the wave nature of light.

(b) Show that any operator A which has no explicit time dependence follows $\frac{d}{dt}\langle A \rangle = \frac{i}{\hbar}\langle [H, A] \rangle$, where

H is the Hamiltonian operator.

(c) A wave function in one-dimension is given by

$$\psi(x) = A\left(1 - x^2\right) : x \in [-1, 1]$$
$$= 0, \quad \text{otherwise}$$

where A is a constant.

Determine the normalization constant A. Also calculate $\langle x \rangle$. (2+1)+4+3

Please Turn Over

3. (a) A particle can be in two different states given by the orthonormal wave functions ψ_1 and ψ_2. If the probability of being in the state ψ_1 is $\frac{1}{3}$, find the normalised wave function of the particle.

   (b) Show that $[AB, C] = A[B, C] + [A, C]B$. Hence find out the value of $\left[\hat{x}, \hat{p}_x^2\right]$.

   (c) Using the commutation relation for the components of the momentum and position operators, show that the components of the angular momentum operator L satisfy $\left[L_x, L_y\right] = i\hbar L_z$.   2+4+4

4. (a) Define a Hermitian operator. Show that the momentum operator is Hermitian.

   (b) Show that if two operators commute, they have common eigenfunctions.

   (c) The parity operator P operates on a function $f(x)$ in the following way : $Pf(x) = f(-x)$. Given that P and the Hamiltonian H commute and $\psi(x)$ is a solution of the time-independent Schrödinger equation, show that $\psi(-x)$ is a solution too with the same eigenenergy as $\psi(x)$. Find out the eigenvalues of the parity operator.   (2+2)+2+(2+2)

5. (a) State the first law of thermodynamics. Using this, prove that for an ideal gas, the relation between C_V and C_P is given by $C_P = C_V + R$ in a quasistatic process where R is the gas constant.

   (b) The equation of state of a hydrostatic system is given by $f(P, V, T) = 0$.

   Prove that $\left(\dfrac{\partial P}{\partial V}\right)_T \left(\dfrac{\partial V}{\partial T}\right)_P \left(\dfrac{\partial T}{\partial P}\right)_V = -1$.

   (c) State Zeroth law of thermodynamics and briefly explain (qualitatively) the concept of temperature from it.   (2+3)+3+2

6. (a) Show that the specific heat at constant volume is related to the second derivative of the Helmholtz free energy.

   (b) For a chemical system, show that $TdS = C_V dT + \dfrac{\beta T}{K_T} dV$, where β and K_T are volume expansivity at constant temperature and isothermal compressibility, respectively.

   (c) Using the fact that dS is an exact differential, derive the following relation :

   $$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P.$$

   Hence show that for a van der Waals gas, the internal energy is not a function of temperature alone.   2+3+5

7. (a) Justify that for an ideal gas, the internal energy is a function of temperature only.

(b) One mole of Nitrogen gas is kept at constant pressure of 1.5 atmos, while its temperature is raised from 127°C to 227°C. Calculate the change in internal energy of the gas. $\left[R = 8.3 \, \mathrm{J\,K^{-1}\,mol^{-1}} \right]$

(c) Derive the relation $C_P = C_V + \left[\left(\dfrac{\partial U}{\partial V} \right)_T + P \right] \left(\dfrac{\partial V}{\partial T} \right)_P$. Let the number of degrees of freedom of a specific gas molecule be f. Then prove that for one mole of this gas, when it behaves like an ideal gas, $C_P = (1 + f/2)R$. 2+3+(2+3)