The matrix form of the standard LPP is

Minimize $Z = Cx$

subject to $Ax = b$

$x \geq 0$

when $c = (-3, 2, 0, 0)$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$, $b = \begin{pmatrix} 14 \\ 6 \end{pmatrix}$,

$A = \begin{pmatrix} -1 & 4 & -1 & 0 \\ -3 & 2 & 0 & 1 \end{pmatrix}$ $\quad 0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

<u>vectors</u>: A matrix containing a single row (or column) is called a row (or column) vector. A vector containing n elements is called an n-component vector

Then matrices $(a_1, a_2, \ldots, a_n)$ and $\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$ are n-component row and column vectors respectively. In the present discussion, we write an n-component column vector as $[a_1, a_2, \ldots a_n]$ and an n-component row vector as $(a_1, a_2, \ldots, a_n)$. The equality, addition and scalar multiplication of two n-component row (or column) vectors are defined as it was defined in matrix.

So, if we take row vectors,

$(a_1, a_2, \ldots a_n) = (b_1, b_2, \ldots, b_n)$ if $a_1 = b_1, a_2 = b_2, \ldots, a_n = b_n$

$(a_1, a_2, \ldots a_n) + (b_1, b_2, \ldots, b_n) = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$

If λ be a scalar, i.e. $\lambda \in \mathbb{R}$. λ is a real number

Then $\lambda (a_1, a_2, \ldots, a_n) = (\lambda a_1, \lambda a_2, \ldots, \lambda a_n)$

If $\alpha = (a_1, a_2, \ldots a_n)$ and $\beta = (b_1, \ldots b_n)$ their scalar product is defined as $\alpha \cdot \beta = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$

null vector $(0, 0, \ldots, 0)$ is called null vector & denoted by θ.

$\mathbb{R}^n = \{(x_1, x_2, \ldots, x_n) : x_1, x_2, \ldots x_n \in \mathbb{R}\}$, the set of all n-component vectors (either row or column) with the addition & scalar multiplication defined earlier is called the Euclidean space of dimension n, $\mathbb{R}$ is the set of all real numbers.

Let $S = \{\alpha_1, \alpha_2, \ldots \alpha_n\} \subset \mathbb{R}^n$. Then S is said to be linearly dependent if there exists n scalars $\lambda_1, \lambda_2, \ldots, \lambda_n \in \mathbb{R}$, not all zero such that $\lambda_1 \alpha_1 + \lambda_2 \alpha_2 + \cdots + \lambda_n \alpha_n = \theta$ where θ is the null vector in $\mathbb{R}^n$

On the other hand if

$$\lambda_1\alpha_1 + \lambda_2\alpha_2 + \cdots + \lambda_n\alpha_n = \theta \text{ implies}$$

$$\lambda_1 = \lambda_2 = \cdots = \lambda_n = 0$$ then the set $S' = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is said to be linearly independent.

Note: 1. A non-empty subset of a linearly independent set is linearly independent

2. A superset of a linearly dependent set is linearly dependent

3. Any set containing the null vector is linearly dependent.

Linear Combination of vectors: A vector α is called a linear combination of the vectors $\alpha_1, \alpha_2, \ldots, \alpha_n$ if α can be written as

$$\alpha = c_1\alpha_1 + c_2\alpha_2 + \cdots + c_n\alpha_n$$ where $c_1, c_2, \ldots, c_n$ are scalars in $\mathbb{R}$.

Line segment in $\mathbb{R}^n$: Let $x = (x_1, x_2, \dots, x_n)$, $y = (y_1, y_2, \dots, y_n)$ be two distinct elements in $\mathbb{R}^n$. Then the set $X = \{x \in \mathbb{R}^n : x = \lambda x + (1-\lambda)y,\ 0 \le \lambda \le 1\}$ is called the line segment joining the points x and y in $\mathbb{R}^n$

Convex set in $\mathbb{R}^n$: A set $S \subset \mathbb{R}^n$ is said to be convex if $x, y \in S$ implies $\lambda x + (1-\lambda)y \in S$ for $0 \le \lambda \le 1$, that is, if two points $x, y \in S$ then the line segment ~~joint~~ joining the points x and y also belongs to S.

Hyperplane: The set $\{(x_1, x_2, \dots, x_n) \in \mathbb{R}^n : c_1x_1 + c_2x_2 + \dots + c_nx_n = z\}$ is called a hyperplane in $\mathbb{R}^n$. We can write the hyper plane as the set $\{x \in \mathbb{R}^n : cx = z\}$ where $c = (c_1, c_2, \dots, c_n)$ $x = [x_1, x_2, \dots, x_n]$, $z \in \mathbb{R}$

Theorem 1 A hyperplane is a convex set in $\mathbb{R}^n$

Let $S = \{x \in \mathbb{R}^n : cx = z\}$ be an hyperplane

Let $x, y \in S$ Then $cx = z$, $cy = z$. Let $0 \le \lambda \le 1$

Then $c(\lambda x + (1-\lambda)y) = c(\lambda x) + c((1-\lambda)y)$.

$= \lambda(cx) + (1-\lambda)cy = \lambda z + (1-\lambda)z = z$

So, $\lambda x + (1-\lambda)y \in S$ for $0 \le \lambda \le 1$

So, S is a convex set. Hence a hyperplane is a convex set

Note: If $\alpha = c_1\alpha_1 + \dots + c_n\alpha_n$ such that $c_1 + c_2 + \dots + c_n = 1$ and $c_i \ge 0$, $i = 1, \dots, n$

then α is said to the convex combination of $\alpha_1, \alpha_2, \ldots, \alpha_n$.

Extreme point : A point x of a convex set X in $\mathbb{R}^n$ is said to be an extreme point if it cannot be expressed as a convex combination two other points in X.

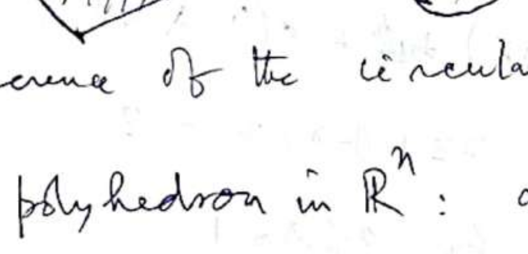
In other words, a point x in a convex set is an extreme point if does not lie on the line segment of any two points, different from x.

Analytically, a point x in a convex set is an extreme point if there do not exist two points x_1 and x_2 $(x_1 \neq x_2)$ in the set such that

$$x = \lambda x_1 + (1-\lambda) x_1 \quad , \quad 0 < \lambda < 1$$

Note that $\lambda \neq 0$ and $\lambda \neq 1$, so that x_1 and x_2 are both different from x.

For example, every vertices of a closed polygon are the extreme points. Similarly, all the points on the



circumference of the circular disc. are extreme points.

Convex polyhedron in $\mathbb{R}^n$: Let $S = \{\alpha_1, \alpha_2, \ldots, \alpha_n\} \subset \mathbb{R}^n$

Then the set $X = \{ x \in \mathbb{R}^n : x = \sum_{i=1}^{n} \lambda_i \alpha_i, \ \lambda_i \geq 0, \ i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} \lambda_i = 1 \}$

is called the convex polyhedron generated by $\alpha_1, \alpha_2, \ldots, \alpha_n$