

2021

ADVANCED BUSINESS MATHEMATICS — HONOURS

Paper : DSE-5.1A

(Module - II)

Full Marks : 40

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer **any four** questions.

1. (a) Evaluate : $\lim_{x \rightarrow 2} \frac{2x^2 - 7x + 6}{3x^2 - 7x + 2}$

(b) A function $f(x)$ is defined as follows : $f(x) = \begin{cases} 2x-1, & \text{if } x < 2 \\ k, & \text{if } x = 2 \\ m-x, & \text{if } x > 2 \end{cases}$

Find the values of k and m if $f(x)$ is continuous at $x = 2$.

5+5

2. (a) Express the following in a single matrix : $\begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 5 \\ 1 & 2 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 7 \\ 3 & 5 & -2 \end{bmatrix}$

(b) Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

5+5

3. (a) Evaluate : $\int \frac{xdx}{(x+1)^2}$

(b) If $x = \frac{1-t}{1+t}$ and $y = \frac{2t}{1+t}$, then show that $\frac{d^2y}{dx^2} = 0$.

5+5

4. (a) Evaluate : $\int \frac{3x}{(x+1)(x-2)} dx$

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(b) Evaluate : $\int \frac{dx}{(x-3)\sqrt{x+1}}$ 5+5

5. (a) Find the area bounded by the parabola $y = x(4 - x)$ and the x -axis.

(b) For a certain establishment, the total cost function C and the total revenue function R are given by $C(x) = x^3 - 12x^2 + 48x + 11$ and $R(x) = 83x - 4x^2 - 21$, where x = output. Obtain the output level for which the profit is maximum and find the maximum profit. 5+5

6. (a) If $y = f(x) = \frac{3x-5}{2x-m}$ and $f(y) = x$, find the value of m .

(b) If the demand function be $p = 40 - \frac{5}{2}q$, find the marginal revenue when the level of output q is 7. 5+5

7. (a) If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$, show that $A^2 = A$. Hence find the matrix B , which satisfies the relation

$3A^2 - 2A + B = I_3$, where I_3 is the identity matrix of order 3.

(b) Prove that $\begin{vmatrix} 1+a_1 & 1 & 1 \\ 1 & 1+a_2 & 1 \\ 1 & 1 & 1+a_3 \end{vmatrix} = a_1 a_2 a_3 \left(1 + \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} \right)$ 5+5

8. (a) Solve by Cramer's Rule :

$x + y + z = 1$, $ax + by + cz = k$, $a^2x + b^2y + c^2z = k^2$, given $a \neq b \neq c$.

(b) If $A^{-1} = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & -1 \end{bmatrix}$, find A . 6+4